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Risk Aversion, Market Power and Credit Spread

Masahiro Ishii Sophia University

Motokazu Ishizaka Chuo University

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KIOICHO, CHIYODA-KU, TOKYO 102-8554, JAPAN

Risk Aversion, Market Power and Credit Spread

Masahiro Ishii
Sophia University *

Motokazu Ishizaka
Chuo University †

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Abstract

To evaluate credit spread, we construct noncooperative games between two lenders, incorporating three factors, namely risk attitude, market power, and incumbent loan portfolio. We consider two cases, in which the loan amount and lending rate are used as strategies, and derive Nash equilibria and reveal the influence of each factor on the spread. Through numerical examples, we find that, in both cases, the greater the lenders' risk aversion, the larger are the spread increments caused by the increase in the correlation coefficient between existing and new loans. (*JEL* D43, D81, G12, G21, L13)

Keywords: credit spread, risk attitude, loan competition, loan portfolio

This study analyzes credit spread, which is defined by the difference between the debt interest rate and the risk-free rate, and aims to determine which factors affect the spread, and how they do so.

The valuation of credit risk has played a central role in corporate finance from both theoretical and practical perspectives. Credit risk evaluation is crucial not only to rational decision-making in lending and borrowing but also to supporting investments by non-financial institutions, thereby promoting economic efficiency. Representative measures for credit risk are the credit spread, credit rating, risky bond price, loss given default, and default probability. There is an extensive body of research on credit risk valuation with academic and practical contributions. In terms of academic contributions, theoretical models have led to an increase in and refinement of asset pricing methods, and provide frameworks for empirical research. Empirical research is useful for identifying and determining influential factors and verifying theoretical models. In terms of practical contributions, it is essential for borrowers to have a clear understanding of the appropriate funding costs. Lenders can set loan amounts and lending rates in accordance

*Faculty of Economics, email: mishii@sophia.ac.jp
†Faculty of Commerce, email: mishizaka@tamacc.chuo-u.ac.jp

with market conditions, and properly quantify credit risk for use in risk management for their own loan portfolios.

Studies on credit risk evaluation can be broadly divided into theoretical models and empirical research. The theoretical models can be further classified into two approaches: an approach that is based on the derivative pricing theory, and an approach that focuses on the lender's decision-making.

We summarize representative studies that belong to the former approach based on the derivative pricing theory, before proceeding to those based on the second approach. This derivative pricing theory approach directs attention toward the structure of borrowers and funding markets, and involves the development of a structural model or a reduced-form model, depending on whether the default occurrence is endogenous or exogenous. The structural model originated with Black and Scholes (1973) and Merton (1974). In this model, default occurrences are endogenously defined based on the fluctuation of the firm's asset value and its capital structure. Merton's well-known definition of default (1974) is that it occurs if the asset value is less than the face value of the liability at maturity. Subsequently, Merton's framework has been expanded in various ways. For instance, models have been proposed that allow defaults before maturity, adopt stochastic interest rate models, and devise expression of default occurrence and default boundaries. Examples include Black and Cox (1976), Kim *et al.* (1993), Nielsen *et al.* (1993), Longstaff and Schwartz (1995), Briys and Varenne (1997), Zhou (2001), and Ishizaka and Takaoka (2003). In the structural model, the factors assumed to influence the credit spread are fluctuations in the firm's asset value, changes in the interest rate, and the definition of default occurrence.

In contrast, the reduced-form model provides the default occurrence exogenously. This model can be considered as more implementable than the structural model because the parameters in the reduced-form model are based on directly observable market data. A representative study is Jarrow and Turnbull (1995), which derives the price curve for risky bonds based on using the HJM model (Heath *et al.* 1992) to describe the term structure of interest rates. Jarrow *et al.* (1997) specifies a time-homogeneous finite state space Markov chain with a generator transition matrix to capture the dynamics of credit rating changes. Duffie and Singleton (1999) uses a hazard rate process to propose a framework for pricing risky bonds that considers both default risk and the interest rate. Bieleck and Rutkowski (2000) and Jarrow *et al.* (2010) deepen the understanding of pricing frameworks that satisfy the no-arbitrage condition by considering both the interest rate and credit risk. Furthermore, from the perspective of practical application, Duffie (1999) and Chiarella *et al.* (2011) develop methods for parameter estimation and numerical calculation. In the above theoretical models, the lender's perspective, behavior, and situation are not considered. Furthermore, in both the structural and reduced-form models, because lenders are assumed to be risk neutral, their risk attitudes are not considered.

The second broad category of theoretical models focuses on the lender's decision-making. A wide variety of models fall under this approach. Boot *et al.* (1991) and Boot and Thakor (1994) show the role of collateral in the presence of borrower's private information. Andersen and Sundaresan (1996) and Fan and Sandaresan (2000) construct a model that considers debt renegotiation and develop a game between lenders and borrowers. Péon and Antelo (2019) derive the impact of information differences among lenders on social welfare under the Cournot model of the loan market. Schargrodsky and Sturzenegger (2000) and Toolsema (2004) apply the Salop model to consider the non-homogeneity of lenders. Under this approach, there are few studies that explicitly set competition among lenders, and lenders are assumed to be risk neutral.

Next, we provide an overview of the empirical research aimed at identifying significant factors that explain credit spread. There are many studies that attempt to extract the key factors explaining spread across various periods and markets. The representative studies are Collin-Dufresne *et al.* (2001), Campbell and Taksler (2003), Longstaff *et al.* (2005), Zhang *et al.* (2009), Tang and Yan (2010), Bao *et al.* (2011), Gilchrist and Zakrejsek (2012), Azad *et al.* (2018), and Wang *et al.* (2020). In those studies, the explanatory factors for credit spread are generally divided into three categories: bond-specific variables, firm-specific variables, and macroeconomic variables. Among the bond-specific variables, the coupon rate, time remaining to maturity, the credit rating, the presence of collateral, and the presence of prepayments are found to be significant, supporting the variables and framework adopted in the theoretical models. In terms of firm-specific variables, the stock price return and volatility, and the asset-liability ratio are identified as significant, and in terms of macroeconomic variables, the interest rate and government bond yields. Of course, the significant factors vary depending on the sample period and market. In recent years, some studies have added market power as an explanatory variable for credit spread. Birchwood *et al.* (2017) and Ornelas *et al.* (2022) utilize the Lerner index. Lian (2018) employs the HHI, and van Leuvensteijin *et al.* (2013) uses the Boone index. However, market power is rarely incorporated into theoretical models of credit risk. Therefore, the theoretical mechanism by which market power affects credit spread is not fully understood. The structure of the spread cannot be generally revealed by the empirical research, unlike the theoretical models.

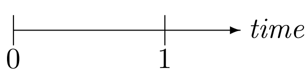
Our review and categorization of the literature on credit risk evaluation indicates that the risk factors themselves and how they affect credit spread have not been sufficiently captured in either the theoretical models or the empirical research. In fact, Collin-Dufresne *et al.* (2001) points out that the factors incorporated in typical models of credit risk cannot explain as much as 25% of the adjusted R-squared. Although they develop additional analyses on sub-samples, adding explanatory variables, such as the government bond yield and the stock price index, the adjusted R-squared does not exceed 0.35. Eom *et al.* (2004) also concludes that it is difficult to accurately predict credit spread using the five structural models. Therefore, based on these results, it is considered that additional factors or a new framework are necessary to increase the explanatory power of the models for credit spread.

Therefore, in this study, we will incorporate three new factors into the model and determine their effects on credit spread. First, competition among lenders is explicitly incorporated into the model. As previously noted, there is a paucity of models in the theoretical studies that explicitly incorporate competition and market power among lenders. Second, we incorporate the lender's risk attitude into the model. Almost all the existing studies assume that the lender is risk neutral. However, as shown empirically by Angelini (2000) and Nishiyama (2007), banks (lenders) are risk averse. Our study includes both risk-neutral and risk-averse lenders. Third, the relationship between existing and new loans is explicitly incorporated into the model. To the best of our knowledge, this feature has not been incorporated into the existing studies. Because lenders such as financial institutions always have existing loans, this relationship should have a significant influence on decision-making regarding new loans. It is difficult to apply the portfolio selection theory to lending activities because lending activities often take time and involve competition from others potential lenders. In practice, this new framework can be a useful tool for the loan portfolio selection problem.

In this study, we establish a noncooperative game of lenders that incorporates these three factors—loan competition, risk attitude, and loan portfolio—and derive the Nash equilibria. Then, we clarify how they appear and influence the credit spread in the Nash equilibrium. We establish two types of noncooperative games, one in which the strategy concerns the loan amount, and the other in which the strategy concerns the lending rate, and compare their Nash equilibria. Our study belongs to the literature regarding the credit risk theoretical model, but we expect our identification of new factors for credit spread to contribute the provision of novel variables for empirical research.

The remainder of the paper is organized as follows. In Section 1, we explain the common assumptions and settings used throughout this study. In Section 2, we derive the Nash equilibrium of the noncooperative game with the loan volume as a strategy, and reveal several characteristics of the lending rate or credit spread. In Section 3, we derive the Nash equilibrium of the noncooperative game with the lending rate as the strategy, and determine the characteristics of credit spread. In Section 4, we discuss the credit spreads in the two Nash equilibria using numerical calculations. Section 5 concludes the paper.

1 Model



In this section, we explain the assumptions and settings that underlie our model. For simplicity, we address the analysis of competition between lenders in a single-period framework. There are two points in time, the beginning of the period *time zero* and the end of the period *time one*.

There is a firm with an initial capital structure that includes only equity. This firm (hereinafter "the firm") is assumed to raise funds by obtaining loans from lenders, and to make an investment using these funds and its existing equity capital. We assume that there are two lenders that are willing to grant a loan to the firm. For convenience, we denote these two lenders by lender j ($j = 1, 2$).

At time 0, the firm borrows funds from the lenders. The lenders accept deposits from savers to finance the loan for the firm. Then, the two lenders compete with each other to offer loans to the firm. At time 1, if the investment succeeds, the firm can repay the principal and interest to the lenders. However, if the investment is unsuccessful, the firm cannot repay the lenders, leading it to default.

1.1 Assumptions and Settings

Let $(\Omega, \mathcal{F}, \mathrm{P})$ be a probability space. For $p \in (0,1)$, let Z_0 be a random variable that has some Bernoulli distribution with parameter $1-p$. At time 0, the firm invests in a project using its equity and debt borrowing from the lenders. An event $\{Z_0 = 1\}$ represents the success of the firm's investment at time 1. Therefore, the probability of success is $1-p$.

Let s, u, and y be nonnegative real numbers. s is the equity capital of the firm at time 0, u is the debt amount, and y is a continuously compound interest rate on that debt u. Then, the sum of the firm's liabilities and net assets is $u+s$, which is equal to the total assets at time 0.

At time 1, the payoff to the shareholders of the firm is

$$(b(u+s)^a - ue^y)\, Z_0,$$

where $0 < a < 1$, $b > 0$. The condition $a > 0$ signifies that augmenting the investment amount results in output increasing, and $a < 1$ denotes the decreasing returns to scale of total assets $u+s$. If the project is successful at time 1, it yields profit $b(u+s)^a$, and the sum of the debt principal and interest ue^y is fully repaid to the lenders. Therefore, the shareholders receive the remainder of the profit once the repayments to the lenders have been made. If the project is unsuccessful, the shareholders receive zero.

For each $y \geq 0$, the firm is assumed to decide on u to maximize the expected payoff to its shareholders, that is,

$$f(u) := \mathrm{E}\left((b(u+s)^a - ue^y)\, Z_0\right) = (1-p)\{b(u+s)^a - ue^y\}.$$

From this assumption, we obtain the firm's demand function for debt as follows:

$$\begin{aligned} & u = \exp\left(\frac{1}{1-a}(-y + \log ab)\right) - s \\ \rightleftharpoons \quad & e^y = ab(u+s)^{-(1-a)} \end{aligned} \tag{1}$$

Now, we explain the assumptions regarding the two lenders. We assume that deposits are elastically supplied at the continuously compound risk-free rate, which is denoted by a positive constant r. Each lender accepts deposits from savers to finance a loan to the firm. For $j = 1, 2$, the amount of money lent by lender j to the firm is denoted by a nonnegative value, u_j. Therefore, the firm's amount of debt is $u = u_1 + u_2$ at time 0. At time 1, lender j will repay $u_j e^r$, which is the deposit principal and the interest on the loan. In addition to granting the loan to the firm, lender j supplies other loans, that is, it has an incumbent loan portfolio. We assume that a random variable Z_j satisfies $\mathrm{E}(Z_j) = 0$ and $\mathrm{V}(Z_j) = 1$, and that the correlation between Z_0 and Z_j is denoted by ρ_j. The random variable Z_j represents the risk factor that affects the profit obtained from the incumbent loan portfolio of lender j. At time 1, lender j receives the profit $\mu_j + \sigma_j Z_j$ from its incumbent loan portfolio, where μ_j and σ_j are positive constants. The profit of lender j at time 1 is expressed by:

$$X_j := u_j e^y Z_0 - u_j e^r + \mu_j + \sigma_j Z_j.$$

On the right-hand side of the equation, the first term $u_j e^y Z_0$ represents the sum of the principal and the interest repaid by the firm at time 1. If the project is successful

($Z_0 = 1$), the lender j will receive $u_j e^y$. However, if the project fails ($Z_0 = 0$), each lender will receive 0.

We have the following objective function of lender j:

$$\mathrm{E}(X_j) - \lambda_j \mathrm{V}(X_j) \tag{2}$$

Here,

$$\begin{aligned} g_j(u_1, u_2) &:= \mathrm{E}(X_j) - \lambda_j \mathrm{V}(X_j) \\ &= -p(1-p)\lambda_j {u_j}^2 e^{2y} + \left\{(1-p) - 2\lambda_j \sigma_j \sqrt{p(1-p)}\rho_j\right\} u_j e^y - u_j e^r \\ &\quad + \mu_j + \lambda_j {\sigma_j}^2 \end{aligned} \tag{3}$$

As y and $u = u_1 + u_2$ satisfy the firm's demand function for debt (1),

$$e^y = ab\,(u_1 + u_2 + s)^{-(1-a)}.$$

From this point forwards, we add the following assumptions.

$$\lambda_1 = \lambda_2 = \lambda, \quad \sigma_1 = \sigma_2 = \sigma, \quad \rho_1 = \rho_2 = \rho. \tag{4}$$

That is, the two lenders are assumed to be homogeneous.

1.2 Lending Competition

In this paper, we derive the Nash equilibria for two cases: one in which the two lenders compete by setting the volume of loans (Cournot competition), and the other in which they compete by setting interest rates for loans (Bertrand competition). Then, we compare the Nash equilibria and clarify the effects of the differences in the means of competing.

First, we establish the former noncooperative game with the loan amout as the strategy. From (1), (3), and (4), we have

$$\begin{aligned} g_j(u_1, u_2) = -A\left\{u_j (u_1 + u_2 + s)^{-(1-a)}\right\}^2 \\ + B u_j (u_1 + u_2 + s)^{-(1-a)} - u_j e^r + \mu_j - \lambda\sigma^2, \end{aligned} \tag{5}$$

where

$$A = a^2 b^2 p(1-p)\lambda, \quad B = ab\{(1-p) - 2\lambda\rho\sigma\sqrt{p(1-p)}\}.$$

Next, we find the Nash equilibrium of the following noncooperative game.

$$\begin{cases} \text{the strategy set for lender } j \text{ is } [0, \infty), \\ \text{the payoff to lender } j \text{ is } g_j(u_1, u_2). \end{cases} \tag{6}$$

Now, we explain a noncooperative game in which two lenders compete with each other using lending interest rates as strategies. In this case, we add the following assumptions regarding the selection of lenders from which the firm receive loans. For the convenience of explanation, we introduce the following functions $v : [0, \infty) \to \mathbf{R}$ and $v_1,\ v_2 : [0, \infty)^2 \to \mathbf{R}$:

$$v(y) = \exp\left(\frac{1}{1-a}\{-y + \log ab\}\right) - s,$$

$$v_1(y_1, y_2) = \begin{cases} v(y_1) & \text{for } y_1 < y_2 \\ \dfrac{1}{2}v(y) & \text{for } y_1 = y_2 = y \\ 0 & \text{for } y_1 > y_2 \end{cases},$$

$$v_2(y_1, y_2) = \begin{cases} 0 & \text{for } y_1 < y_2 \\ \dfrac{1}{2}v(y) & \text{for } y_1 = y_2 = y \\ v(y_2) & \text{for } y_1 > y_2 \end{cases}.$$

Here, $y_j \geq 0$ represents the lending rate offered by lender j to the firm. If the lending rate offered by lender 1 is y_1 and that offered by lender 2 is y_2, then the amount borrowed by the firm from lender 1 is expressed as $v_1(y_1, y_2)$. $v_2(y_1, y_2)$ represents the loan amount of lender 2 to the firm in this situation. This means that the firm borrows funds from the lender offering the lower lending rate of the two lenders.

In addition, we assume that $\hat{g}_j(y_1, y_2) = g_j(v_1(y_1, y_2), v_2(y_1, y_2))$ for each $j = 1, 2$. Then, we derive the Nash equilibrium of the following noncooperative game.

$$\begin{cases} \text{the strategy set for lender } j \text{ is } \left[0, \log \dfrac{ab}{s^{1-a}}\right], \\ \text{the payoff to lender } j \text{ is } \hat{g}_j(y_1, y_2). \end{cases} \tag{7}$$

We derive the Nash equilibrium of (6) in Section 2, and that of (7) in Section 3.

2 Competition with the loan amount as the strategy.

In this section, we explain the Nash equilibrium of the noncooperative game with the loan volume as the strategy (6), and the procedure for its derivation. In addition, we explain the properties of the lending rate in the Nash equilibrium.

Theorem 1 shows the Nash equilibrium of the noncooperative game (6). Before proceeding to the theorem, we state the lemmas necessary to prove Theorem 1 in order.

Lemma 1
If $B \leq 0$, g_1 is maximized at $u_1 = 0$ for any $u_2 \geq 0$. A similar situation holds for g_2.

Lemma 2
When $\frac{1}{2} \leq a < 1$, $\frac{\partial g_1}{\partial u_1}(u_1, u_2)$ is decreasing with u_1 for any $u_2 \geq 0$. A similar situation holds for $\frac{\partial g_2}{\partial u_2}$.

Hereafter, we assume that $\frac{1}{2} \leq a < 1$, and that $K = \{w \geq 0 \mid (w + s)^{1-a} e^r < B\}$.

Lemma 3
Let $B > s^{1-a}e^r$. For each $w \geq 0$, we define the following equation with an unknown u:

$$-2A\frac{u(au+w+s)}{(u+w+s)^{3-2a}} + B\frac{au+w+s}{(u+w+s)^{2-a}} - e^r = 0.$$

This equation has a unique solution, which is denoted by $\varphi(w)$ hereafter.

Lemma 4

(i) If $B \leq s^{1-a}e^r$, g_1 is maximized at $u_1 = 0$ for any $u_2 \geq 0$.

(ii) If $B > s^{1-a}e^r$, for any $u_2 \in K$, there exists a unique value of $u_1 > 0$ such that $\dfrac{\partial g_1}{\partial u_1}(u_1, u_2) = 0$. Then, g_1 is maximized at that point.

A similar situation holds for g_2.

Lemma 5
Let $B > s^{1-a}e^r$. The best response set of lender 1 is $D_1 = \{(\varphi(u_2), u_2) \,|\, u_2 \in K\}$, and the best response set of lender 2 is $D_2 = \{(u_1, \varphi(u_1)) \,|\, u_1 \in K\}$.

Lemma 6
Let $B > s^{1-a}e^r$. There exists a unique positive solution for the following equation with an unknown u:

$$-2A\frac{u\{(1+a)u+s\}}{(2u+s)^{3-2a}} + B\frac{(1+a)u+s}{(2u+s)^{2-a}} - e^r = 0. \tag{8}$$

Hereafter, the solution is denoted by $\psi(\lambda)$.

Lemma 7
Let $B > s^{1-a}e^r$. The following system of two equations with unknowns u_1 and u_2 has exactly one solution.

$$\begin{cases} u_1 = \varphi(u_2) \\ u_2 = \varphi(u_1) \end{cases}$$

Then, we can write the solution as $(u_1, u_2) = (\psi(\lambda), \psi(\lambda))$.

According to Lemmas 1 to 7 above, we can find the Nash equilibrium of the noncooperative game (6). We state this as the following theorem.

Theorem 1
If $B > s^{1-a}e^r$, the Nash equilibrium of the noncooperative game (6) is

$$(u_1, u_2) = (\psi(\lambda), \psi(\lambda)). \tag{9}$$

Otherwise, the Nash equilibrium is $(u_1, u_2) = (0, 0)$.

Assuming that $B > s^{1-a}e^r$, the Nash equilibrium is (9) from Theorem 1. Then, by using the firm's demand function for debt (1), we obtain

$$e^y = ab(2\psi(\lambda) + s)^{-(1-a)}.$$

Thus, the lending rate in the Nash equilibrium is

$$y = \log\left(ab(2\psi(\lambda) + s)^{-(1-a)}\right). \tag{10}$$

Here, when $\lambda = 0$, that is, when the lenders are risk neutral, the following result is obtained.

Corollary 2
In the case that the lenders are risk neutral, that is, $\lambda = 0$, the lending rate in the Nash equilibrium (10) is

$$y = r + \log\frac{1}{1-p} + \log\frac{2\psi(0) + s}{(1+a)\psi(0) + s}. \tag{11}$$

In equation (11), the loan spread is the sum of the second and third terms on the right-hand side of (11).

If the default probability of a debt with face value 1 is p, the value of the debt at time 0 is $e^{-r}(1-p)$ under the risk-neutral valuation method. Therefore, the lending rate y on the debt is

$$y = r + \log\frac{1}{1-p}. \tag{12}$$

In this case, from the comparison of (11) and (12), the increase in the loan spread caused by imperfect competition between the two lenders is

$$\log\frac{2\psi(0) + s}{(1+a)\psi(0) + s} > 0. \tag{13}$$

The third term on the right-hand side of the lending rate (11) in the Nash equilibrium when lenders are risk neutral, that is, (13), is considered to be caused by the market power of lenders.

Corollary 3
For $\lambda > 0$, that is, for risk-averse lenders, the Nash equilibrium lending rate (10) is decomposed as

$$y = r + \log\frac{1}{1-p} + \log\frac{2\psi(0) + s}{(1+a)\psi(0) + s} + (1-a)\log\frac{2\psi(0) + s}{2\psi(\lambda) + s}. \tag{14}$$

In the decomposition of the Nash equilibrium lending rate (14), the loan spread is the sum of the second, third, and fourth terms on the right-hand side.

Under the assumption that lenders are risk neutral ($\lambda = 0$), the noncooperative game with the loan amount as the strategy (6) can be interpreted as a loan version of the so-called Cournot model. The fourth term on the right-hand side of the decomposition of the Nash equilibrium lending rate (14) reflects the risk attitude of lenders. The corresponding factor does not appear in the standard Cournot model.

Corollary 4
When $p = 0$, that is, when the default probability of the firm is equal to 0, the Nash equilibrium lending rate (10) is

$$y = r + \log\frac{2\psi(0) + s}{(1+a)\psi(0) + s}. \tag{15}$$

Even if the default probability of the firm is zero, the loan spread exists because of imperfect competition among the lenders, i.e., it results from their market power.

Corollary 5
For $\rho \geq 0$, the following (a) and (b) hold.

(a) $\psi(0) \geq \psi(\lambda)$. Then,

$$\log \frac{2\psi(0)+s}{2\psi(\lambda)+s} \geq 0$$

in the decomposition of the Nash equilibrium lending rate (14), and the loan spread is nonnegative.

(b) $\dfrac{\partial \psi}{\partial \lambda}(\lambda) < 0$.

In other words, if the correlation coefficient ρ between the existing portfolio profit $\mu + \sigma Z_j$ and the success of the firm's investment Z_0 is nonnegative, the higher is the risk aversion of the lenders, the lower is the loan amount in the Nash equilibrium.

Corollary 6
For $\lambda > 0$, $\rho \geq 0$ leads to $\dfrac{\partial \psi}{\partial \sigma}(\lambda) \leq 0$, and $\rho < 0$ leads to $\dfrac{\partial \psi}{\partial \sigma}(\lambda) > 0$.

When $\rho \geq 0$, the Nash equilibrium loan amount decreases as the variance of the lender's incumbent portfolio profit σ increases. On the other hand, when $\rho < 0$, it increases as σ increases.

3 Competition with lending rate as the strategy.

In this section, we derive the Nash equilibrium of the noncooperative game with the lending rate as the strategy (7). Then, we explain the relationships between the lending rate in the Nash equilibrium and other parameters.

Theorem 2 shows the Nash equilibrium of the noncooperative game (7). We will first state several lemmas to prove this Theorem.

Lemma 8
If $B \leq 0$, $\hat{g}_1$ is maximized at $y_1 = \log \dfrac{ab}{s^{1-a}}$ for any $y_2 > y_1$. A similar situation holds for $\hat{g}_2$.

Lemma 9
When $\dfrac{1}{2} \leq a < 1$, $\frac{\partial \hat{g}_1}{\partial y_1}(y_1, y_2)$ is increasing with y_1 for any $y_2 > y_1$. A similar situation holds for $\dfrac{\partial \hat{g}_2}{\partial y_2}$.

Hereafter, we assume that $\dfrac{1}{2} \leq a < 1$.

Lemma 10
(i) If $B \leq s^{1-a} e^r$, $\hat{g}_1$ is maximized at $y_1 = \log \dfrac{ab}{s^{1-a}}$ for any $y_2 > y_1$.

(ii) If $B > s^{1-a}e^r$, for any $y_2 > y_1$, there exists a unique y_1 such that $\dfrac{\partial \hat{g}_1}{\partial y_1}(y_1, y_2) = 0$. Then, $\hat{g}_1$ is maximized at that point. This unique solution is denoted by $\hat{y}_1$.

A similar situation holds for $\hat{g}_2$.

Lemma 11

(i) If $B \leq s^{1-a}e^r$, $\hat{g}_1(y, y)$ is maximized at $y = \log \dfrac{ab}{s^{1-a}}$.

(ii) If $B > s^{1-a}e^r$, there exists a unique y such that $\dfrac{\partial \hat{g}_1}{\partial y}(y, y) = 0$. Then, $\hat{g}_1$ is maximized at that point. This unique solution is denoted by $\tilde{y}$.

The same holds true for $\hat{g}_2(y, y)$.

Lemma 12

$\tilde{y} < \hat{y}_j$, for $j = 1, 2$.

Let $\bar{g}(y)$ be defined as follows.

$$\bar{g}(y) = -A\{v(y)(v(y) + s)^{-(1-a)}\}^2 + Bv(y)(v(y) + s)^{-(1-a)} - v(y)e^r + \mu - \lambda\sigma^2 \quad (16)$$

Lemma 13

If $B > s^{1-a}e^r$, there exists a unique y such that $\hat{g}_1(y, y) = \bar{g}(y)$. This unique solution is denoted by $\bar{y}$. The same holds true for $\hat{g}_2(y, y)$.

Based on Lemmas 8 to 13 above, we can find the Nash equilibrium of the noncooperative game (7). We state this as the following theorem.

Theorem 7

(i) If $B \leq s^{1-a}e^r$, the Nash equilibrium of the noncooperative game (7) is $(y_1, y_2) = \left(\log \frac{ab}{s^{1-a}}, \log \frac{ab}{s^{1-a}}\right)$.

(ii) If $B > s^{1-a}e^r$, we have the following.

(a) If $\bar{g}(0) \geq \hat{g}_1(0, 0)$, the Nash equilibrium of the noncooperative game (7) is $(y_1, y_2) = (0, 0)$.

(b) If $\bar{g}(0) < \hat{g}_1(0, 0)$, the Nash equilibrium of the noncooperative game (7) is $(y_1, y_2) = \{(y_1, y_2) | \max(0, \dot{y}) \leq y_1 = y_2 \leq \bar{y}\}$,
where $\dot{y}$ is a unique solution to the equation $\bar{g}(y) = \mu - \lambda\sigma^2$.

Corollary 8

In the case that the lenders are risk neutral, that is, $\lambda = 0$, the lending rate in the Nash equilibrium of the noncooperative game (7) is

$$(y_1, y_2) = \left(r + \log \frac{1}{1-p}, r + \log \frac{1}{1-p}\right). \quad (17)$$

This lending rate of the Nash equilibrium equals (12), that is, the interest rate that reflects only default risk. The difference from the case where the loan amount is the strategy in Section 2 is that the term relating to market power does not appear.

Corollary 9
Let C, D, and F be defined as follows.

$$C = \frac{1}{2\lambda\sigma\sqrt{p(1-p)}},\ D = \lambda p(1-p)v(0),\ F = e^r - 1 + p.$$

For $\lambda > 0$ and $B > s^{1-a}e^r$, we have the following.

(i) If $\rho < -C\left(\frac{3}{2}D + F\right)$, the Nash equilibrium of the noncooperative game (7) is $(y_1, y_2) = (0, 0)$.

(ii) If $-C\left(\frac{3}{2}D + F\right) \leq \rho < -C\left(\frac{1}{2}D + F\right)$, the Nash equilibrium of the noncooperative game (7) is $(y_1, y_2) = \{(y_1, y_2)|0 \leq y_1 = y_2 \leq \bar{y}\}$.

(iii) If $-C\left(\frac{1}{2}D + F\right) \leq \rho \leq 1$, the Nash equilibrium of the noncooperative game (7) is $(y_1, y_2) = \{(y_1, y_2)|\dot{y} \leq y_1 = y_2 \leq \bar{y}\}$.

Corollary 9 is a rewrite of Theorem 7 from the perspective of the correlation coefficient between existing and new loans. It states that as the correlation coefficient becomes more strongly negative, the range of the Nash equilibrium shifts downward. In other words, when making a decision on a new loan, lenders may tolerate lower lending rates if there is negative correlation between the returns on new and existing loans.

Corollary 10
For $\lambda > 0$ and $B > s^{1-a}e^r$, we have the following.

(i) If $\rho < -\frac{3pe^r v^2\left(r + \log\frac{1}{1-p}\right)}{4\sigma\sqrt{p(1-p)}}$, then $\bar{y} < r + \log\frac{1}{1-p}$.

(ii) If $\rho < -\frac{3\lambda p(1-p)v^2\,(r)}{4\lambda\sigma\sqrt{p(1-p)}}$, then $\bar{y} < r$.

In addition to Corollary 9, Corollary 10 suggests that the lending rate of the Nash equilibrium may sometimes be lower than the rate reflecting only default (12) or even lower than the risk-free rate in cases where the correlation coefficient between existing and new loans is extremely negative.

4 Discussions

In sections 2 and 3, we described some properties of the credit spread in the Nash equilibria of (6) and (7), respectively. In this section, we employ numerical examples to compare the credit spreads of the Nash equilibria for each type of competition. By using numerical examples, richer insights into market performance can be gained.

The credit spread in the Nash equilibrium of the noncooperative game with the loan amount as the strategy (6) is the sum of the second, third, and fourth terms in equation (14), that is,

$$\log\frac{1}{1-p}+\log\frac{2\psi(0)+s}{(1+a)\psi(0)+s}+(1-a)\log\frac{2\psi(0)+s}{2\psi(\lambda)+s}. \tag{18}$$

On the other hand, the minimum and maximum values of the credit spread in the Nash equilibrium of the noncooperative game with the lending rate as the strategy (7) are as follows, respectively:

$$\dot{y}-r, \tag{19}$$

$$\bar{y}-r. \tag{20}$$

We compare the values of these credit spreads. Then, based on the comparison, we examine the impact of the lenders' risk attitude, market power, and the incumbent loan portfolios on the credit spreads of new loans.

First, a, b, p, s, r , and σ are given as follows:

$$a=\frac{3}{4},\quad b=2,\quad p=\frac{1}{200},\quad s=1,\quad r=\frac{1}{20},\quad \sigma=\frac{3}{100}.$$

Then, we compute (18), (19), and (20) for each $\lambda=\frac{1}{1000},\frac{1}{500},\frac{1}{300}$.

The range of the correlation coefficient ρ is set to $[-\frac{9}{10},\frac{9}{10}]$, and the graph of equation (18) for each λ is presented in Figure 1. This figure shows that the lender requires a larger spread as it becomes more risk averse. In addition, it is evident that all credit spreads are increasing with respect to the correlation coefficient. This fact implies that lenders will accept smaller spreads when the correlation coefficient ρ between new loans and incumbent loans is negative.

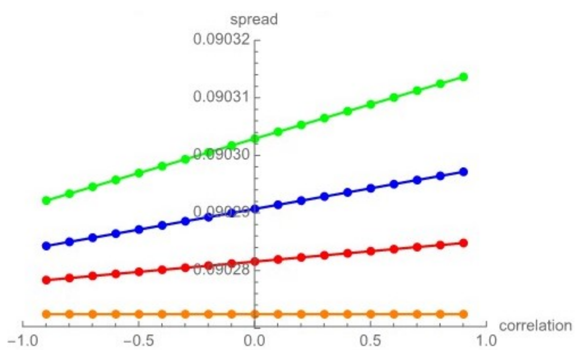
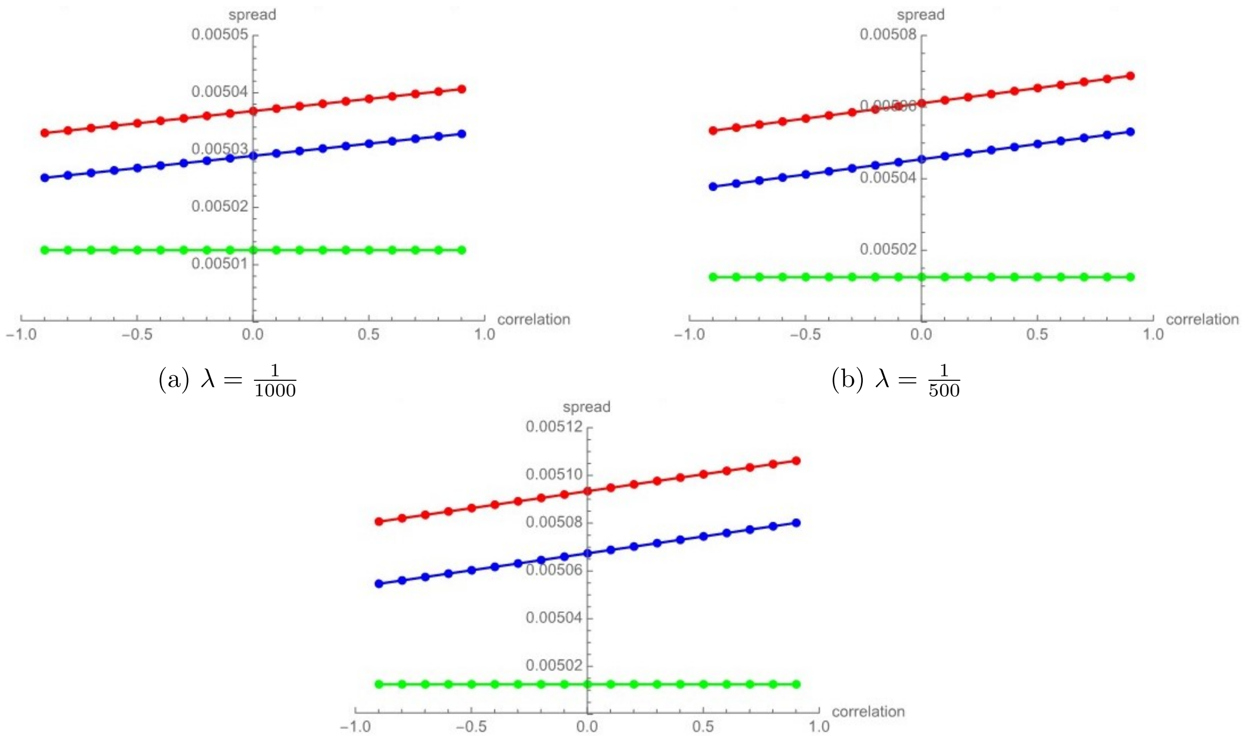


Figure 1: Credit Spreads in Loan Amount Competition

Note: The green line shows the credit spread for $\lambda=\frac{1}{300}$, the blue line that for $\lambda=\frac{1}{500}$, the red line that for $\lambda=\frac{1}{1000}$, and the orange shows the credit spread for $\lambda=0$ (risk neutral).

Next, setting the range of the correlation coefficient ρ to $[-\frac{9}{10},\frac{9}{10}]$, we present a set of graphs for each λ of Equations (19) and (20) in Figures 2(a), 2(b), and 2(c). The graph for $\lambda = \frac{1}{1000}$ is presented in Figure 2(a), that for $\lambda = \frac{1}{500}$ in Figure 2(b), and that for $\lambda = \frac{1}{300}$ in Figure 2(c). In contrast with the standard Bertrand competition model, the lending rate (price) is greater than the risk-free rate (marginal cost). In other words, a positive credit spread is observed. Furthermore, when the lending rate is the strategy, similar to the credit spread in the Nash equilibrium of the noncooperative game where the loan amount is the strategy, an increase in the lender's risk aversion will require a larger spread. It is evident that lenders are willing to accept smaller spreads when the correlation coefficient ρ between new loans and incumbent loans is negative.



(a) $\lambda = \frac{1}{1000}$

(b) $\lambda = \frac{1}{500}$

(c) $\lambda = \frac{1}{300}$

Figure 2: Credit Spreads with Lending Rate Competition

Note: The red line denotes the maximum credit spread $\bar{y} - r$, the blue line denotes $\dot{y} - r$, and the green denotes $\lambda = 0$ (risk neutral).

For any (λ, ρ), the value of the credit spread (18) is much larger than that of (20). This implies that the different strategies adopted in the competition among lenders, i.e., the competition with the loan amount as the strategy or the one with lending rate as the strategy, have a significant impact on the market performance. This suggests that it is necessary to consider the type of competition among lenders in empirical analyses aimed at finding the determinants of credit spreads. For any type of competition, the larger is the risk aversion λ, the larger is the increment of the credit spread caused by the increase of correlation coefficient ρ.

In summary, the above analysis reveals some of the effects that three factors, i.e., risk aversion, market power, and the relationship with incumbent loan portfolios, have on credit spreads, and the manner in which they are affected.

5 Conclusion

In this paper, we propose an analytical framework to investigate the effects of the lenders' risk attitudes, market power, and incumbent loan portfolios on credit spread in a competitive market. We develop two noncooperative game models in which two lenders compete. Loan amounts are the strategies on which they compete in one game, and lending rates are the strategies in the other. This paper contributes to the literature by (i) considering the impact of lender risk aversion on credit spreads, (ii) incorporating the influence of incumbent loan portfolios into new loan decisions, and (iii) extending the Cournot and Bertrand models to loan competition under uncertainty. In general, it is difficult to analyze the impact of economic agents' attitudes toward risk on market performance in empirical analyses. However, the use of theoretical models makes it possible to investigate the effects of such latent variables and provides hypotheses for empirical analysis.

We find that the credit spread in equilibrium varies significantly depending on the type of competition. When competing based on the loan amount as a strategy, even if lenders are risk neutral, the credit spread will increase not only because of the default risk but also because of market power. Conversely, when the strategy is the lending rate, if the lenders are risk neutral, the credit spread in equilibrium is caused only by the default risk of the firm. If lenders are risk averse, their attitude toward risk will be reflected in the credit spread in equilibrium in any form of competition. Then, the degree of risk aversion, the variance of the incumbent loan portfolio value, and the correlation coefficient with the new loan all affect the equilibrium credit spreads. The findings from this study suggest some variables that may be convincing in empirical analyses of the factors determining credit spreads, and hypotheses how the variables affect spreads. In addition, our results have important implications for the risk management of loan portfolios for lenders in a competitive environment, as well as for the relationships between product portfolio and capital structure of non-financial companies.

Further research could be undertaken in the following directions. First, we should analytically investigate the impact of the relationship between the new loan and the incumbent loan portfolio on credit spreads in more detail. Next, a natural extension of the model is to consider the case where there are n lenders in the loan market. Third, an additional future extension is to examine the case where non-homogeneous lenders compete.

References

[1] Anderson, R. W., and S. M. Sundaresan. 1996. Design and Valuation of Debt Contracts. *Reveiw of Financial Studies.* 9(1):37–68.

[2] Angelini, P. 2000. Are Banks Risk Averse? Intraday Timing of Operations in the Interbank Market. *Journal of Money Credit and Banking.* 32(1):54–73.

[3] Azad, A. S. M. S., A. Chazi, P. Cooper, and A. Ahsan. 2018. What Determines the Japanese Corporate Credit Spread? A New Evidence. *Research in International Business and Finance*, 45:349–356.

[4] Bao, J., J. Pan, and J. Wang. 2011. The Illiquidity of Corporate Bonds. *Journal of Finance*, 66(3):911–946.

[5] Bielecki, T. R., and M. Rutkowski. 2000. Multiple Rating Model of Defaultable Term Structure. *Mathematical Finance*. 10(2):125–139.

[6] Birchwood, A., M. Brei, and D. M. Noel. 2017. Interest margins and bank regulation in Central America and the Caribbean. *Journal of Banking and Finance*. 85:56–68.

[7] Black, F., and J. C. Cox. 1976. Valuing Corporate Securities: Some Effects of Bond Indenture Provisions. *Journal of Finance*. 31:351–367.

[8] Black, F., and M. Scholes. 1973. The Pricing of Options and Corporate Liabilities. *Journal of Political Economy*. 81(3):637–654.

[9] Boot, A. W. A., and A. V. Thakor. 1994. Moral Hazard and Secured Lending in an Infinitely Repeated Credit Market Game. *International Eonomic Review*. 35(4):899–920.

[10] Boot, A. W. A., A. V. Thakor, and G. F. Udell. 1991. Secured Lending and Default Risk: Equilibrium Analysis, Policy Implications and Empirial Results. *Economic Journal*. 101:458–472.

[11] Briys, E., and F. D. Varenne. 1997. Valuing Risky Fixed Rate Debt: An Extension. *Journal of Financial and Quantitative Analysis*. 32(2):239–248.

[12] Campbell, J. Y., and G. B. Taksler. 2003. Equity Volatility and Corporate Bond Yields. *Journal of Finance*. 58(6):2321–2349.

[13] Chiarella, C., V. Fanelli, and S. Musti. 2011. Modelling the Evolution of Credit Spreads Using the Cox Process within the HJM Framework: A CDS Option Pricing Model. *European Journal of Operational Research*. 208:195–218.

[14] Collin-Dufresne, P., R. S. Goldstein, and J. S. Martin. 2001. The Determinants of Credit Spread Changes. *Journal of Finance*. 56(6):2177–2207.

[15] Duffie, G. R. 1999. Estimating the Price of Default Risk. *Review of Financial Studies*. 12(1):197–226.

[16] Duffie, D., and K. J. Singleton. 1999. Modeling Term Structures of Defaultable Bonds. *Review of Financial Studies*. 12(4):687–720.

[17] Eom, Y. H., J. Helwege, and J-Z. Huang. 2004. Structural Models of Corporate Bond Pricing: An Empirical Analysis. *Review of Financial Studies*. 17(2):499–544.

[18] Fan, H., and S. M. Sundaresan. 2000. Debt Valuation, Renegotiation, and Optimal Dividend Policy. *Review of Financial Studies*. 13(4):1057–1099.

[19] Gilchrist, S., and E. Zakrajšek. 2012. Credit Spreads and Business Cycle Fluctuations. *American Economic Reveiw*. 102(4):1692–1720.

[20] Heath, D., R. Jarrow, and A. Morton. 1992. Bond Pricing and the Term Structure of Interest Rates: A New Methodology for Contingent Claims Valuation. *Econometrica*. 60:77–105.

[21] Ishizaka, M., and K. Takaoka. 2003. On the Pricing of Defaultable Bonds Using the Framework of Barrier Options. *Asia-Pacific Financial Markets*. 10(2-3):151–162.

[22] Jarrow, R. A., D. Lando, and S. M. Turnbull. 1997. A Markov Model for the Term Structure of Credit Risk Spreads. *Review of Financial Studies*. 10(2):481–523.

[23] Jarrow, R. A., D. Lando, and F. Yu. 2005. Default Risk and Diversification: Theory and Empirical Implications. *Mathematical Finance*. 15(1):1–26.

[24] Jarrow, R. A., H. Li, S. Liu, and C. Wu. 2010. Reduced-form Valuation of Callable Corporate Bonds: Theory and Evidence. *Journal of Financial Economics*. 95(2):227–248.

[25] Jarrow, R. A., and S. M. Turnbull. 1995. Pricing Derivatives on Financial Securities Subject to Credit Risk. *Journal of Finance*. 50(1):53–85.

[26] Kim, J., K. Ramaswamy, and S. Sundaresan. 1993. Does Default Risk in Coupons Affect the Valuation of Corporate Bonds? A Contingent Claims Model. *Financial Management*. 22(3):117–131.

[27] Leuvensteijn, M. van, C. K. Sϕrensen, J. A. Bikker, and A. A. R. J. M. van Rixtel. 2013. Impact of Bank Competition on the Interest Rate Pass-through in the Euro Area. *Applied Economics*. 45:1359–1380.

[28] Lian, Y. 2018. Bank Competition and the Cost of Bank Loans. *Review of Quantitative Finance and Accounting*. 51(1):253–282.

[29] Longstaff, F. A., S. Mithal, and E. Neis. 2005. Corporate Yield Spreads: Default Risk of Liquidity? New Evidence from the Credit Default Swap Market. *Journal of Finance*. 60(5):2213-2253.

[30] Longstaff, F. A., and E. S. Schwartz. 1995. A Simple Approach to Valuing Risky Fixed and Floating Rate Debt. *Journal of Finance*. 50(3):789–819.

[31] Merton, R. 1974. On the Pricing Corporate Debt: The Risk Structure of Interest Rate. *Journal of Finance*. 29(2):449–470.

[32] Nielsen, L. T., J. Saà -Requejo, and P. Santa-Clarra. 1993. Default Risk and Interest Rate Risk: The Term Structure of Default Spreads. Working Paper, INSEAD, Fontainebleau, France.

[33] Nishiyama, Y. 2007. Are Banks Risk-Averse? *Eastern Economic Journal*. 33(4):471–490.

[34] Ornelas, J. R. H., M. S. da Silva, and B. F. N. van Doormik. 2022. Informational Switching Costs, Bank Competition, and the Cost of Finance. *Journal of Banking and Finance*. 138, Article No.106408.

[35] Peón, D., and M. Antelo. 2019. Do Bad Borrowers Hurt Good Borrowers? A Model of Biased Banking Competition. *Portuguese Economic Journal*. 18:5–17.

[36] Salop, S. 1979. Monopolistic Competition with Outside Goods. *Bell Journal of Economics*. 10(1):141–156.

[37] Schargrodsky, E., and F. Sturzenegger. 2000. Banking Regulation and Competition with Product Differentiation. *Journal of Development Economics*. 63:85–111.

[38] Tang, D. Y., and H. Yan. 2010. Market Conditions, Default Risk and Credit Risk. *Journal of Banking and Finance* 34(4):743–753.

[39] Toolsema, L. A. 2004. Monetary Policy and Market Power in Banking. *Jounal of Economics*. 83(1):71–83.

[40] Wang, C-W., W-C. Chiu, and T-H. D. King. 2020. Debt Maturity and the Cost of Bank Loans. *Journal of Banking and Finance*. 112:1–22.

[41] Zhang, B. Y., H. Zhou, and H. Zhu. 2009. Explaining Credit Default Swap Spreads with the Equity Volatility and Jump Risk of Individual Firms. *Review of Financial Studies*. 22(12):5099–5131.

[42] Zhou, C. 2001. The Term Structure of Credit Spreads with Jump Risk. *Journal of Banking and Finance*. 25(11):2015–2040.