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Risk Aversion, Market Power and Credit Spread

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Risk Aversion, Market Power and Credit Spread

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Abstract

To analyze credit spreads, we extend both Cournot and Bertrand competition models between lenders, incorporating three factors: risk attitude, market power, and current loan portfolio. This study contributes by (i) revealing the simultaneous influences of these three factors on credit spreads, (ii) demonstrating the differential impact on spreads resulting from two types of market competition, and (iii) uncovering the relation between existing and new loan risks on spreads. We examine two market competition models: in Cournot, each lender strategically chooses the loan amount, while in Bertrand, lenders compete with lending rates as the strategy. In the latter model, it is found that the Bertrand paradox in economics is resolved due to the risk aversion of lenders. Furthermore, it is shown that the credit spread in the Nash equilibrium of the Bertrand competition is greater than that of the Cournot competition under certain conditions, which cannot be observed in the case of risk-neutral lenders. The results obtained from the model can explain the market outcome of credit spreads in the actual debt market. This study also offers new perspectives and hypotheses for empirical research.

Keywords: credit spread, risk attitude, loan competition, loan portfolio
JEL Classification Numbers: D43, D81, G12, G21, L13

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1 Introduction

This study analyzes credit spread, which is defined by the difference between the debt interest rate and the risk-free rate, and aims to determine which factors affect the spread, and how they do so.

The valuation of credit risk has played a central role in corporate finance from both theoretical and practical perspectives. Credit risk evaluation is crucial not only to rational decision-making in lending and borrowing but also to supporting investments by non-financial institutions, thereby promoting economic efficiency. Representative measures for credit risk are the credit spread, credit rating, risky bond price, loss given default, and default probability. There is an extensive body of research on credit risk valuation with academic and practical contributions. In terms of academic contributions, theoretical models have led to an increase in and refinement of asset pricing methods, and provide frameworks for empirical research. Empirical research is useful for identifying and determining influential factors and verifying theoretical models. In terms of practical contributions, it is essential for borrowers to have a clear understanding of the appropriate funding costs. Lenders can set loan amounts and lending rates in accordance with market conditions, and properly quantify credit risk for use in risk management for their own loan portfolios.

Studies on credit risk evaluation can be broadly divided into theoretical models and empirical research. The theoretical models can be further classified into two approaches: an approach that is based on the derivative pricing theory, and an approach that focuses on the lender's decision-making.

We summarize representative studies that belong to the former approach based on the derivative pricing theory, before proceeding to those based on the second approach. This derivative pricing theory approach directs attention toward the structure of borrowers and funding markets, and involves the development of a structural model or a reduced-form model, depending on whether the default occurrence is endogenous or exogenous. The structural model originated with Black and Scholes (1973) and Merton (1974). In this model, default occurrences are endogenously defined based on the fluctuation of the firm's asset value and its capital structure. Merton's well-known definition of default (1974) is that it occurs if the asset value is less than the face value of the liability at maturity. Subsequently, Merton's framework has been expanded in various ways. For instance, models have been proposed that allow defaults before maturity, adopt stochastic interest rate models, and devise expression of default occurrence and default boundaries. Examples include Black and Cox (1976), Kim *et al.* (1993), Nielsen *et al.* (1993), Longstaff and Schwartz (1995), Briys and Varenne (1997), Zhou (2001), and Ishizaka and Takaoka (2003). In the structural model, the factors assumed to influence the credit spread are fluctuations in the firm's asset value, changes in the interest rate, and the definition of default occurrence.

In contrast, the reduced-form model provides the default occurrence exogenously. This model can be considered as more implementable than the structural model because the parameters in the reduced-form model are based on directly observable market data. A representative study is Jarrow and Turnbull (1995), which derives the price curve for risky bonds based on using the HJM model (Heath *et al.* 1992) to describe the term structure of interest rates. Jarrow *et al.* (1997) specifies a time-homogeneous finite state space Markov chain with a generator transition matrix to capture the dynamics of credit rating changes. Duffie and Singleton (1999) uses a hazard rate process to propose a framework for pricing risky bonds that considers both default risk and the interest rate. Bieleck and Rutkowski (2000) and Jarrow *et al.* (2010) deepen the understanding of pricing frameworks that satisfy the no-arbitrage condition by considering both the interest rate and credit risk. Furthermore, from the perspective of practical application, Duffie (1999) and Chiarella *et al.* (2011) develop methods for parameter estimation and numerical calculation. In the above theoretical models, the lender's perspective, behavior, and situation are not considered. Furthermore, in both the structural and reduced-form models, because lenders are assumed to be risk neutral, their risk attitudes are not considered.

The second broad category of theoretical models focuses on the lender's decision-making. A wide variety of models fall under this approach. Boot *et al.* (1991) and Boot and Thakor (1994) show the role of collateral in the presence of borrower's private information. Andersen and Sundaresan (1996) and Fan and Sandaresan (2000) construct a model that considers debt renegotiation and develop a game between lenders and borrowers. Péon and Antelo (2019) derive the impact of information differences among lenders on social welfare under the Cournot model of the loan market. Schargrodsky and Sturzenegger (2000) and Toolsema (2004) apply the Salop model (1979) to consider the non-homogeneity of lenders. Under this approach, there are few studies that explicitly set competition among lenders, and lenders are assumed to be risk neutral.

Next, we provide an overview of the empirical research aimed at identifying significant factors that explain credit spread. There are many studies that attempt to extract the key factors explaining spread across various periods and markets. The representative studies are Collin-Dufresne *et al.* (2001), Campbell and Taksler (2003), Longstaff *et al.* (2005), Zhang *et al.* (2009), Tang and Yan (2010), Bao *et al.* (2011), Gilchrist and Zakrejsek (2012), Azad *et al.* (2018), and Wang *et al.* (2020). In those studies, the explanatory factors for credit spread are generally divided into three categories: bond-specific variables, firm-specific variables, and macroeconomic variables. Among the bond-specific variables, the coupon rate, time remaining to maturity, the credit rating, the presence of collateral, and the presence of prepayments are found to be significant, supporting the variables and framework adopted in the theoretical models. In terms of firm-specific variables, the stock price return and volatility, and the asset-liability ratio are identified as significant, and in terms of macroeconomic variables, the interest rate and government bond yields. Of course, the significant factors vary depending on the sample period and market. In recent years, some studies have added market power as an explanatory variable for credit spread. Birchwood *et al.* (2017) and Ornelas *et al.* (2022) utilize the Lerner index. Lian (2018) employs the HHI, and van Leuvensteijin *et al.* (2013) uses the Boone index. However, market power is rarely incorporated into theoretical models of credit risk. Therefore, the theoretical mechanism by which market power affects credit spread is not fully understood. The structure of the spread cannot be generally revealed by the empirical research, unlike the theoretical models.

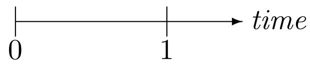
Our review and categorization of the literature on credit risk evaluation indicates that the risk factors themselves and how they affect credit spread have not been sufficiently captured in either the theoretical models or the empirical research. In fact, Collin-Dufresne *et al.* (2001) points out that the factors incorporated in typical models of credit risk cannot explain as much as 25% of the adjusted R-squared. Although they develop additional analyses on sub-samples, adding explanatory variables, such as the government bond yield and the stock price index, the adjusted R-squared does not exceed 0.35. Eom *et al.* (2004) also concludes that it is difficult to accurately predict credit spread using the five structural models. Therefore, based on these results, it is considered that additional factors or a new framework are necessary to increase the explanatory power of the models for credit spread.

Therefore, in this study, we will incorporate three new factors into the model and determine their effects on credit spread. First, competition among lenders is explicitly incorporated into the model. As previously noted, there is a paucity of models in the theoretical studies that explicitly incorporate competition and market power among lenders. Second, we incorporate the lender's risk attitude into the model. Almost all the existing studies assume that the lender is risk neutral. However, as shown empirically by Angelini (2000) and Nishiyama (2007), banks (lenders) are risk averse. Our study includes both risk-neutral and risk-averse lenders. Third, the relationship between existing and new loans is explicitly incorporated into the model. To the best of our knowledge, this feature has not been incorporated into the existing studies. Because lenders such as financial institutions always have existing loans, this relationship should have a significant influence on decision-making regarding new loans. It is difficult to apply the portfolio selection theory to lending activities because lending activities often take time and involve competition from others potential lenders. In practice, this new framework can be a useful tool for the loan portfolio selection problem.

In this study, we establish a noncooperative game of lenders that incorporates these three factors—loan competition, risk attitude, and loan portfolio—and derive the Nash equilibria. Then, we clarify how they appear and influence the credit spread in the Nash equilibrium. We establish two types of noncooperative games, one in which the strategy concerns the loan amount (Cournot competition), and the other in which the strategy concerns the lending rate (Bertrand competition), and compare their Nash equilibria. Our study belongs to the literature regarding the credit risk theoretical model, but we expect our identification of new factors for credit spread to contribute the provision of novel variables for empirical research.

The remainder of the paper is organized as follows. In section 2, we explain the common assumptions and settings used throughout this study. In Section 3 and 4, we derive the Nash equilibrium of the noncooperative game with the loan volume as a strategy and that with the lending rate as a strategy, respectively, and reveal several characteristics of credit spreads. In Section 5, we provide several numerical examples to illustrate the spreads in Nash equilibria. In Section 6, we discuss by directly comparing the lending rates in the Nash equilibria derived form two types of market competitions. Section 7 concludes the paper.

2 Model



In this section, we explain the assumptions and settings that underlie our model. For simplicity, we address the analysis of competition between lenders in a single-period framework. There are two points in time, the beginning of the period *time zero* and the end of the period *time one*.

There is a firm with an initial capital structure that includes only equity. This firm (hereinafter "the firm") is assumed to invest in a project using both funds obtained through debt financing and its own equity. We assume that there are n homogeneous lenders that are willing to grant a loan to the firm. For convenience, we denote these n lenders by lender j ($j = 1, 2, \dots, n$).

At time 0, the firm borrows funds from the lenders. The lenders accept deposits from savers to finance the loan for the firm. Then, the n lenders compete with each other to offer loans to the firm. At time 1, if the project succeeds, the firm can repay the principal and interest to the lenders. However, if the project is unsuccessful, the firm cannot repay the lenders, leading it to default.

Next, we will explain the assumptions and settings in detail. Let $(\Omega, \mathcal{F}, \mathrm{P})$ be a probability space. For $p \in (0, 1)$, let Z_0 be a random variable taking the values 0 and 1 with probability p and $1 - p$. At time 0, the firm invests in a project using its own equity and debt borrowing from the lenders. An event $\{Z_0 = 1\}$ represents the success of the firm's project, and the probability of success is $1 - p$. In other words, at time 0, only the probability of the project's success is known, and at time 1, the outcome of the project becomes clear.

Let s, u, and y be nonnegative real numbers. s is the equity capital of the firm at time 0, u is the debt amount, and y is a continuously compound interest rate on that debt u. Then, the sum of the firm's liabilities and net assets is $u + s$, which is equal to the total assets at time 0.

At time 1, the payoff to the shareholders of the firm is

$$(b(u+s)^a - ue^y)\, Z_0,$$

where $0 < a < 1$ and $b > 0$. The condition $a > 0$ indicates that an increase in invested capital leads to an increase in revenue, and the condition $a < 1$ means that the law of diminishing returns to scale of total assets $u + s$ is in effect. If the project is successful at time 1, it yields profit $b(u + s)^a$, and the sum of the debt principal and interest ue^y is fully repaid to the lenders. Therefore, the shareholders receive the remainder of the profit once the repayments to the lenders have been made. If the project is unsuccessful, the shareholders receive zero.

For each $y \geq 0$, the firm is assumed to decide on u to maximize the expected payoff to its shareholders, that is,

$$f(u) := \mathrm{E}\left((b(u+s)^a - ue^y)\, Z_0\right) = (1-p)\{b(u+s)^a - ue^y\}.$$

From this assumption, we obtain the firm's demand function for debt as follows:

$$u = v(y) = \exp\left(\frac{1}{1-a}(-y + \log ab)\right) - s \tag{1}$$

It apparent that v is a one-to-one correspondence, the relation between the debt amount u and the debt interest rate y, (1), is written as

$$e^y = ab(u+s)^{-(1-a)}.$$

Now, we explain the assumptions regarding the n homogeneous lenders. We assume that deposits are elastically supplied at the continuously compound risk-free rate, which is denoted by a positive constant r. Each lender accepts deposits from savers to finance a loan to the firm. For $j = 1, 2, \ldots, n$, the amount of money lent by lender j to the firm is denoted by a nonnegative value, u_j. Therefore, the firm's amount of debt is $u = \sum_{k=1}^{n} u_k$ at time 0. At time 1, lender j will repay $u_j e^r$, which is the deposit principal and the interest on the loan. In addition to granting the loan to the firm, lender j supplies other loans, that is, it has a current loan portfolio. We assume that a random variable Z_j satisfies $\mathrm{E}(Z_j) = 0$ and $\mathrm{V}(Z_j) = 1$, and that the correlation between Z_0 and Z_j is denoted by ρ. The random variable Z_j represents the risk factor that affects the profit obtained from the current loan portfolio of lender j. At time 1, the value of Z_j becomes clear, and lender j receives the profit $\mu + \sigma Z_j$ from its current loan portfolio, where μ and σ are positive constants. The profit of lender j at time 1 is expressed by:

$$X_j := u_j e^y Z_0 - u_j e^r + \mu + \sigma Z_j$$

On the right-hand side of the equation, the first term $u_j e^y Z_0$ represents the sum of the principal and the interest repaid by the firm at time 1. If the project is successful ($Z_0 = 1$), the lender j will receive $u_j e^y$. However, if the project fails ($Z_0 = 0$), each lender will receive 0.

We assume that $E(X_j) - \lambda \mathrm{V}(X_j)$ is the objective function of lender j at time 0, where a non-negative constant λ denotes the risk avers of each bank.

Here,

$$\begin{aligned} g_j(u_1, u_2, \ldots, u_n) &:= \mathrm{E}(X_j) - \lambda \mathrm{V}(X_j) \\ &= -p(1-p)\lambda {u_j}^2 e^{2y} + \left\{(1-p) - 2\lambda\sigma\sqrt{p(1-p)}\rho\right\} u_j e^y - u_j e^r + \mu - \lambda\sigma^2, \end{aligned} \tag{2}$$

As y and $u = \sum_{k=1}^{n} u_k$ satisfy the firm's demand function for debt (1),

$$e^y = ab\left(\sum_{k=1}^{n} u_k + s\right)^{-(1-a)}. \tag{3}$$

3 Non-cooperative game with lending amount as a strategy

In this section, we derive the Nash equilibrium for the case in which n lenders compete by setting the volume of loans (Cournot competition), and investigate the lending rate in the Nash equilibrium. First, we describe a noncooperative game with the loan amount as the strategy.

From (2) and (3), we have

$$
\begin{aligned}
& g_j(u_1, u_2, \ldots, u_n) \\
&= -A\left\{u_j\left(\sum_{k=1}^{n} u_k + s\right)^{-(1-a)}\right\}^2 + Bu_j\left(\sum_{k=1}^{n} u_k + s\right)^{-(1-a)} - u_j e^r + \mu - \lambda\sigma^2 \qquad (4)
\end{aligned}
$$

where

$$A = a^2b^2p(1-p)\lambda, \quad B = ab\left\{(1-p) - 2\lambda\rho\sigma\sqrt{p(1-p)}\right\}.$$

Then, we find the Nash equilibrium of the following noncooperative game.

$$
\begin{cases}
\text{the strategy set for lender } j \text{ is } [0, \infty), \\
\text{the payoff to lender } j \text{ is } g_j(u_1, u_2, \ldots, u_n).
\end{cases} \qquad (5)
$$

The noncooperative game (5) represents that n lenders strategically choose a volume of loan to maximize the objective function at time 0. That is, (5) is a Cournot competition.

Theorem 1 shows the Nash equilibrium of the noncooperative game (5) .

Theorem 1
Let $\frac{1}{2} \leq a < 1$.

(i) If $B \leq s^{1-a}e^r$ holds, the unique Nash equilibrium of (5) is

$$(u_1, u_2, \ldots, u_n) = (0, 0, \ldots, 0). \qquad (6)$$

(ii) Otherwise, that is, $B > s^{1-a}e^r$ holds, the unique Nash equilibrium of (5) is

$$(u_1, u_2, \ldots, u_n) = (\psi(n,\lambda), \psi(n,\lambda), \ldots, \psi(n,\lambda)), \qquad (7)$$

where $\psi(n,\lambda)$ is the unique solution of the following equation with unknown u,

$$-2A \cdot \frac{u\{(n-1+a)u + s\}}{(nu+s)^{3-2a}} + B \cdot \frac{(n-1+a)u + s}{(nu+s)^{2-a}} - e^r = 0. \qquad (8)$$

See Appendix for the proof [1] .

Hereafter, we assume that $\frac{1}{2} \leq a < 1$ and $B > s^{1-a}e^r$ in this section. According to the Nash equilibrium (7) and the firm's demand function for debt (3),

$$e^y = ab(n\psi(n,\lambda) + s)^{-(1-a)}.$$

[1] This paper omits the proofs of all theorems and corollaries. If you are interested in the proofs, please contact the author by email.

Then, the lending rate in the Nash equilibrium is

$$\xi(n,\lambda) := \log\left(ab(n\psi(n,\lambda)+s)^{-(1-a)}\right). \tag{9}$$

The following corollary is obtained from (9).

Corollary 2
The lending rate in the Nash equilibrium (9) is decomposed as follows:

$$\xi(n,\lambda) = r + \log\frac{1}{1-p} + \log\frac{n\psi(n,0)+s}{(n-1+a)\psi(n,0)+s} + (1-a)\log\frac{n\psi(n,0)+s}{n\psi(n,\lambda)+s}. \tag{10}$$

Corollary 3
If the probability that the project fails is equal to 0, that is, $p=0$, the decomposition of $\xi(n,\lambda)$ is as follows:

$$\xi(n,\lambda) = r + \log\frac{n\psi(n,0)+s}{(n-1+a)\psi(n,0)+s}.$$

Based on the above two corollaries, It becomes clear that what causes the each component of $\xi(n,\lambda)$. The third term of (10) does not depend on λ, and is positive. Even if the probability of project failure is 0, the third term of (10) in $\xi(n,\lambda)$ still exists. Therefore, the third term represents the market power arising from imperfect competition among the finite lenders. It is a decreasing function with respect to a. Therefore, it is considered that an increase in the capital efficiency of the borrowing company acts to reduce the spread. On the other hand,the forth term of (10) is equal to 0 when the lenders are risk neutral, that is, $\lambda=0$. Then, the forth term reflects the risk aversion of lenders. Thus, the impact of market power and risk attitude on spreads is clarified.

Corollary 4
For $\rho \geq 0$, we have $\psi(n,0) \geq \psi(n,\lambda)$, which implies

$$\log\frac{n\psi(n,0)+s}{n\psi(n,\lambda)+s} \geq 0,$$

and

$$\frac{\partial\psi}{\partial\lambda}(n,\lambda) < 0.$$

The first part of Corollary 4 and (10) give us that the spread $\xi(n,\lambda)-r$ is non-negative for $\rho \geq 0$. According to the second inequality, the more risk averse lenders are, the lower the amount of lending and the higher the lending rate in Nash equilibrium.

Corollary 5
When $\lambda > 0$, $\frac{\partial\psi}{\partial\sigma}(n,\lambda) \leq 0$ for $\rho \geq 0$, and $\frac{\partial\psi}{\partial\sigma}(n,\lambda) > 0$ for $\rho < 0$.

Corollary 5 shows the effect of current loans on the lending rates of new loans. If a new borrower is a start-up company whose business differs from incumbent industries, this corollary implies that active lending to that company is beneficial for the lender's entire loan portfolio. Furthermore, it suggests that the relationship between current loans and new loans must be taken into account to investigate the composition factors of spreads in empirical analysis.

Theorem 6
The sequence of lending rates in the Nash equilibrium $\{\xi(n,\lambda)\}_{n=2,3,\ldots}$ converges as $n \to \infty$, and

$$\lim_{n\to\infty} \xi(n,\lambda) = r + \log \frac{1}{1-p} + \log \frac{1-p}{1-p-2\lambda\sigma\rho\sqrt{p(1-p)}}. \tag{11}$$

Hereafter, we put $\xi = \lim_{n\to\infty} \xi(n,\lambda)$.

Theorem 6 implies that as the number of lenders increases, the third term in the lending rate $\xi(n,\lambda)$ that reflects market power converges to 0 as $n \to \infty$. On the other hand, the component of the spread that arises due to the risk attitude of lenders remains even if the number of lenders increases. In particular, when $\rho < 0$, the lending spread is smaller than in the risk-neutral case.

For the convenience, we put the spread in (11) as

$$\zeta := \xi - r = \log \frac{1}{1-p} + \log \frac{1-p}{1-p-2\lambda\sigma\sqrt{p(1-p)}\rho}.$$

Corollary 7
For $\rho > 0$, we have $\zeta > \log \frac{1}{1-p}$. On the other hand, we have $\zeta < \log \frac{1}{1-p}$ for $\rho < 0$.

Corollary 8
(a) For $\rho > 0$, ζ is an increasing function of σ, where $0 < \sigma < \frac{1}{2\lambda\rho}\sqrt{\frac{1-p}{p}}$.

(b) For $\rho < 0$, ζ is a decreasing function of σ.

Corollary 9
(a) For $\rho > 0$, ζ is an increasing function of p, where $0 < p < \frac{1}{1+4\lambda^2\sigma^2\rho^2}$.

(b) For $\rho < 0$, ζ is minimized at $p = \frac{1}{2} - \frac{1}{2\sqrt{1+4\lambda^2\sigma^2\rho^2}}$.

Corollary 9 suggests that for new businesses and startups with risk profiles distinct from those of incumbent industries, a reduction in its risk will not necessarily lead to a decrease in debt financing costs.

4 Non-cooperative game with lending rate as a strategy

In this section, we derive the Nash equilibrium of the noncooperative game where the lending rate is the strategy (Bertrand competition), and provide an analysis of the lending rate under this equilibrium.

To derive the Nash equilibrium, we introduce some functions used in this section in a sequential manner.

To begin with, for each $j = 1,2,\ldots,n$, $u_j : [0,\infty)^n \to [0,\infty)$ is defined as follows.

$$u_j(y_1,y_2,\ldots,y_n) = \begin{cases} 0 & \text{if } \exists k = 1,2,\ldots,n, \quad y_k < y_j \\ \frac{1}{m}v(y) & \text{if } \exists k = 1,2,\ldots,m-1 \in \{1,2,\ldots,j-1,j+1,\ldots,n\}, \\ & \quad y_j = y_{k_1} = y_{k_2} = \cdots = y_{k_{m-1}} = y, \text{and} \\ & \quad \forall k \in \{1,2,\ldots,n\} \setminus \{j,k_1,k_2,\ldots,k_{m-1}\}, \quad y_k > y \end{cases} \tag{12}$$

u_j represents the loan amount provided by lender j. If any other lender offers a lower rate than lender j, the loan amount by lender j becomes zero. If m lenders including lender j offer the lowest rate, the total loan amount is equally divided among them.

Next, for each $m = 1.2. \ldots$, $G_m : \mathbf{R} \to \mathbf{R}$ is defined as follows.

$$\begin{aligned} G_m(y) &= -A\left[\frac{1}{m}v(y)\{v(y)+s\}^{-(1-a)}\right]^2 + \frac{B}{m}v(y)\{v(y)+s\}^{-(1-a)} \\ &\quad - \frac{1}{m}v(y)e^r + \mu - \lambda\sigma^2 \end{aligned} \tag{13}$$

G_m describes the profit when the total loan amount is equally divided among m lenders who offer the lowest lending rate y.

Finally, for each $j = 1.2. \ldots, n$, $\hat{g}_j : \left(-\infty, \log \frac{ab}{s^{1-a}}\right] \to \mathbf{R}$ is defined as follows.

$$\hat{g}_j(y_1, y_2, \ldots, y_n) = \begin{cases} \mu - \lambda\sigma^2 & \text{if} \quad \exists k = 1, 2, \ldots, n, \quad y_k < y_j \\ G_m(y) & \text{if} \quad \exists k = 1, 2, \ldots, m-1 \in \{1, 2, \ldots, j-1, j+1, \ldots, n\}, \\ & \qquad y_j = y_{k_1} = y_{k_2} = \cdots = y_{k_{m-1}} = y, \text{and} \\ & \qquad \forall k \in \{1, 2, \ldots, n\} \setminus \{j, k_1, k_2, \ldots, k_{m-1}\}, \quad y_k > y \end{cases} \tag{14}$$

$\hat{g}_j$ represents the profit of lender j. If any other lender offers a lower rate than lender j, j does not provide any loans, and thus its profit comes only from the existing loan. Otherwise, the profit of lender j is G_m defined by equation (13)

Then, we formulate the following non-cooperattie game.

$$\begin{cases} \text{the strategy set for lender } j \text{ is } \left(-\infty, \log \dfrac{ab}{s^{1-a}}\right], \\ \text{the payoff to lender } j \text{ is } \hat{g}_j(y_1, y_2, \ldots, y_n). \end{cases} \tag{15}$$

After several lemmas, we derive the following theorem.

Theorem 10
Let $\frac{1}{2} \leq a < 1$.

(i) If $B \leq s^{1-a}e^r$, the Nash equilibrium of the noncooperative game (15) is as follows.

$$y_1 = y_2 = \cdots = y_n = \log \frac{ab}{s^{1-a}}. \tag{16}$$

(ii) If $B > s^{1-a}e^r$, the Nash equilibrium of the noncooperative game (15) is as follows.

$$\left\{(y_1, y_2, \ldots, y_n) = (y, y, \ldots, y) \,|\, \underline{\xi}(n, \lambda) \leq y \leq \bar{\xi}(n, \lambda)\right\}. \tag{17}$$

where $\underline{\xi}(n, \lambda)$ is a unique solution to the following equation with respect to y,

$$-\frac{A}{m} \cdot \frac{v(y)}{(v(y)+s)^{2(1-a)}} + B \cdot \frac{1}{(v(y)+s)^{1-a}} - e^r = 0. \tag{18}$$

And $\bar{\xi}(n, \lambda)$ is a unique solution to the following equation with respect to y,

$$-\frac{(m+1)A}{m} \cdot \frac{v(y)}{(v(y)+s)^{2(1-a)}} + B \cdot \frac{1}{(v(y)+s)^{1-a}} - e^r = 0. \tag{19}$$

Equation (17) in Theorem 10 shows that the Bertrad paradox in economics are resolved when lenders are risk reverse.

Corollary 11
In the case that the lenders are risk neutral, that is, $\lambda = 0$, the Nash equilibrium of the noncooperative game (15) is as follows.

$$y_1 = y_2 = \cdots = y_n = r + \log \frac{1}{1-p}. \tag{20}$$

In the case of risk neutral, the spread refletcts only the defaut risk of the borrower. Therefore, the term caused by the market power observed in the Cournot competiton does not appear. Therefore, the spread in the Bertrand competiton depends only on risk attitude.

The following two corollaries describe the limits of the upper and lower bounds of the lending rate (17) in the Nash equilibrium.

Corollary 12
Let $\frac{1}{2} \leq a < 1$ and $B > s^{1-a}e^r$.
$\{\bar{\xi}(n,\lambda)\}_{n=2,3,\ldots}$ is a monotonically decreasing sequence bounded below. Then, it converges. We denote the limit of this sequence by $\bar{\xi}$.

Corollary 13
Let $\frac{1}{2} \leq a < 1$ and $B > s^{1-a}e^r$.
$\{\underline{\xi}(n,\lambda)\}_{n=2,3,\ldots}$ is a monotonically decreasing sequence bounded below. Then, it converges. We denote the limit of this sequence by $\underline{\xi}$.

5 Numerical examples

In sections 3 and 4, we described some properties of the credit spread in the Nash equilibria of (5) and (15), respectively. In this section, we employ numerical examples in case of $n = 2$. By using numerical examples, richer insights into market performance can be gained.

When $n = 2$, the credit spread in the Nash equilibrium of the noncooperative game with the loan amount as the strategy (5) is the sum of the second, third, and fourth terms in equation (10), that is,

$$\log \frac{1}{1-p} + \log \frac{2\psi(0)+s}{(1+a)\psi(0)+s} + (1-a)\log \frac{2\psi(0)+s}{2\psi(\lambda)+s}. \tag{21}$$

On the other hand, the minimum and maximum values of the credit spread in the Nash equilibrium of the noncooperative game with the lending rate as the strategy (15) are as follows, respectively:

$$\bar{\xi}(2,\lambda) - r, \tag{22}$$

$$\underline{\xi}(2,\lambda) - r. \tag{23}$$

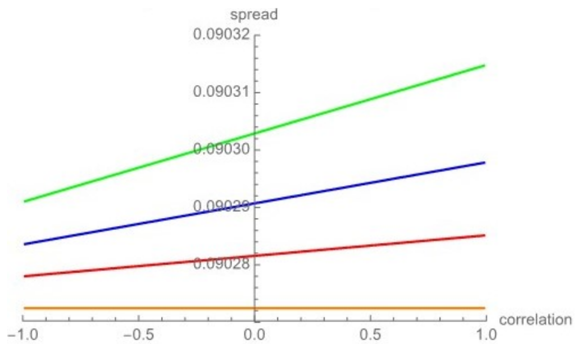


Figure 1: Credit Spreads in Loan Amount Competition

Note: The green line shows the credit spread for $\lambda = \frac{1}{300}$, the blue line that for $\lambda = \frac{1}{500}$, the red line that for $\lambda = \frac{1}{1000}$, and the orange shows the credit spread for $\lambda = 0$ (risk neutral).

We compare the values of these credit spreads. Then, based on the comparison, we examine the impact of the lenders' risk attitude, market power, and the incumbent loan portfolios on the credit spreads of new loans.

First, a, b, p, s, r , and σ are given as follows:

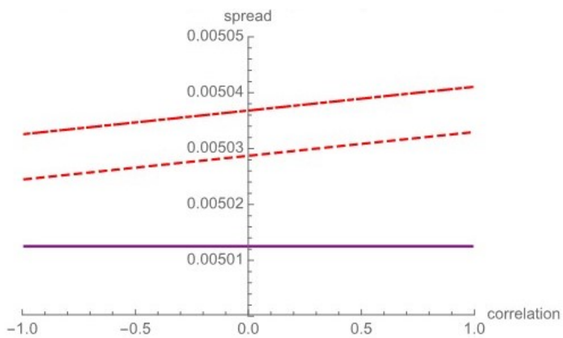
$$a = \frac{3}{4}, \quad b = 2, \quad p = \frac{1}{200}, \quad s = 1, \quad r = \frac{1}{20}, \quad \sigma = \frac{3}{100}.$$

Then, we compute (21), (22), and (23) for each $\lambda = \frac{1}{1000}, \frac{1}{500}, \frac{1}{300}$.

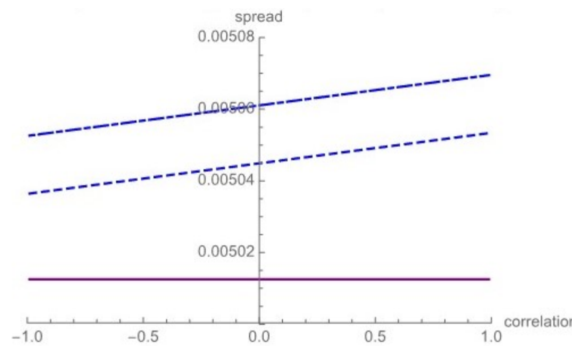
The range of the correlation coefficient ρ is set to $[-\frac{99}{100}, \frac{99}{100}]$, and the graph of equation (21) for each λ is presented in Figure 1. This figure shows that the lender requires a larger spread as it becomes more risk averse. In addition, it is evident that all credit spreads are increasing with respect to the correlation coefficient. This fact implies that lenders will accept smaller spreads when the correlation coefficient ρ between new loans and incumbent loans is negative.

Next, setting the range of the correlation coefficient ρ to $[-\frac{99}{100}, \frac{99}{100}]$, we present a set of graphs for each λ of Equations (22) and (23) in Figures 2(a), 2(b), and 2(c). The graph for $\lambda = \frac{1}{1000}$ is presented in Figure 2(a), that for $\lambda = \frac{1}{500}$ in Figure 2(b), and that for $\lambda = \frac{1}{300}$ in Figure 2(c). In contrast with the basic Bertrand competition model, the lending rate (price) is greater than the risk-free rate (marginal cost). In other words, a positive credit spread is observed. Furthermore, when the lending rate is the strategy, similar to the credit spread in the Nash equilibrium of the Cournot competition, an increase in the lender's risk aversion will require a larger spread. It is evident that lenders are willing to accept smaller spreads when the correlation coefficient ρ between new loans and incumbent loans is negative.

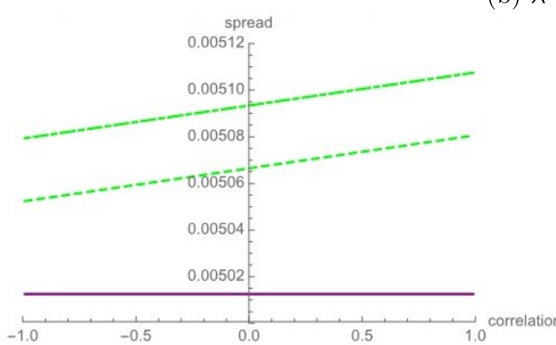
In these numerical examples, the value of the credit spread (21) is much larger than that of (23) for any (λ, ρ). This implies that the different strategies adopted in the competition among lenders, i.e., the competition with the loan amount as the strategy or the one with lending rate as the strategy, have a significant impact on the market performance. Howevere, as shown in the next section, this realtionship is reversed under



(a) $\lambda = \frac{1}{1000}$



(b) $\lambda = \frac{1}{500}$



(c) $\lambda = \frac{1}{300}$

Figure 2: Credit Spreads in Lending Rate Competition

Note: The dash-dot line denotes the maximum credit spread $\bar{\xi}(2,\lambda)-r$, the dashed line denotes $\underline{\xi}(2,\lambda)-r$, and the purple denotes $\lambda = 0$ (risk neutral).

certain condition. In any case, it follows that it is necessary to consider the type of competition among lenders in empirical analyses aimed at finding the determinants of credit spreads. For any type of competition, the larger is the risk aversion λ, the larger is the increment of the credit spread caused by the increase of correlation coefficient ρ.

In summary, the above analysis reveals some of the effects that three factors, i.e., risk aversion, market power, and the relationship with incumbent loan portfolios, have on credit spreads, and the manner in which they are affected. Furthermore, a direct comparison of lending rates form two competitons is presented in the next section.

6 Discussions

In this section, we compare the lending rate in the Nash equilibrium under loan amount competition derived in Section 3 with that under lending rate competition in Section 4. Here, it is worth confirming that, under the basic model, the price in the Nash equilibrium with Cournot competition is always higher than that with Bertrand competition. Hereafter, in this section, we refre to loan amount competition as Counot competition and lendeng rate competition as Bertrand competition.

Corollary 14
For $n = 2, 3, \ldots$, the following inequality holds.

$$\underline{\xi}(n, \lambda) < \xi(n, \lambda). \tag{24}$$

This corollary implies that the lending rate in the Nash equilibrium with Cournot competition is always higher than the lower bound of the lending rate with Bertrand competition, when the number of lenders is finite.

The following proposition is one of the most unique and significant results we have obtained.

Proposition 15
If the following inequality holds for $n = 2, 3, \ldots$,

$$(n-1)A \geq e^r \frac{(1-a)\left\{n\psi(n,\lambda)+s\right\}^{2(1-a)}}{(n-1+a)\psi(n,\lambda)+s}, \tag{25}$$

then,

$$\xi(n, \lambda) < \bar{\xi}(n, \lambda). \tag{26}$$

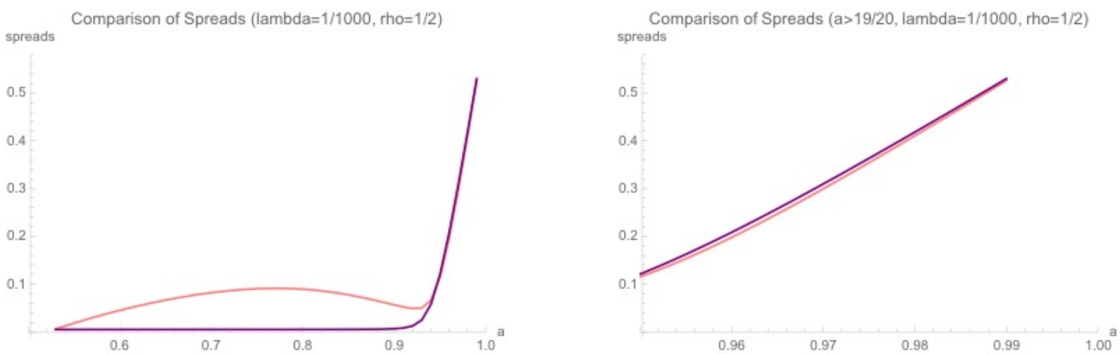
This proposition suggests that there is a reversal in the relatioship that holds between the price in the Nash equilibrium with Cournot competition and that with Bertrand competition in the basic model. Namely, when inequality (25) holds, the upper bound of the lending rate in the Nash Equilibrium with Bertrand competition exceeds the lending rate with Cournot competition. Furthermore, as stated in the following proposition, the lending rate in the Nash Equilibrium with Bertrand is always higher in the limit case, even without requiring the condition equivalent to inequality (25).

Proposition 16
As $n \to \infty$, the following holds.

$$\underline{\xi} = \xi < \bar{\xi}. \tag{27}$$

To illustrate this reversal, we attempt to verify Proposition 15 using numerical example for the case of $n = 2$. Specifically, this numerical example demonstrates that the reversal of lending rates is caused by the variation in the value of a. This is varied from $\frac{53}{100}$ to $\frac{99}{100}$ in increments of $\frac{1}{100}$. The values for the other parameters are as follows.

$$b = 2,\ p = \frac{1}{200},\ \lambda = \frac{1}{1000},\ s = 1,\ r = \frac{1}{20},\ \sigma = \frac{3}{100},\ \rho = \frac{1}{2}.$$



(a) Spreads corresponding to the value of a

(b) Enlarged view of (a)

Figure 3: Comparison of spreads

In Figure 3, the magenta curve shows the spreads in the Nash equilibrium with Cournot competition, while the purple curve represents Bertrand competiton case. The graph on the right (b) is a enlarged view of the region near $a = 1$ in the graph (a). Indeed, near $a = 1$, inequality (25) is satisfied and a reversal of the lending rates is observed.

7 Conclusion

In this paper, we propose an analytical framework to investigate the effects of the lenders' risk attitudes, market power, and incumbent loan portfolios on credit spread in a competitive market. Two types of non-cooperative games are constructed for the lender: one is the Cournot competition in which the strategy is the loan amount and the other is the Bertrand competition in which the strategy is the lending rate. The Nash equilibria for both cases are derived, together with the decomposition of the lending rates, an analysis of their properties in the limiting case, and comparison of two cases.

Through the decomposition of lending rates in the Nash equilibrium and several numerical examples, the impact of the three focal factors highlighted above on the credit spread is revealed. In the Cournot, it is shown that how each factor influences the loan portfolio and the equilibrium lending rate is determined solely by the risk attitude under the limiting case.

In the Bertrand, it was revealed that the equilibrium lending rate inherently reflects only the risk attitude, and that the Bertrand paradox is resolved. This is caused through two pathways. One is the correlation coefficient between the payoff of the incumbent loan portfolio and that of new loans. The higher the correlation, the smaller the risk brought by new loans, and the lender acts accordingly. Therefore, the equilibrium lending interest rate increases. The other pathway is the risk generated from new loan themselves. A decrease in the lending rate leads to an increase in the loan amount, that is, an increase in risk for the lenders. Therefore, when the lending rate falls below a certain level, lending by multiple lenders rather than a single lender results in a greater objective function value. Due to these two effects, a lending rate higher than that of the risk-neutral case is included in the Nash equilibrium

Furthermore, we identify a reversal — absent in the basic model — where the lending rate in the Nash equilibrium with Bertrand competition surpasses that with Cournot competition, along with the conditions under which this reversal arises. An increase in the number of lenders causes the maximum of the Nash equilibrium in the Bertrand model to become larger than the equilibrium lending rate in the Cournot model. The increase in the number of lenders reduces the impact of the correlation between new loans and the incumbent loan portfolio to zero. However, the disadvantage of offering a lower lending rate than competitors and taking on all loans to the firm by a single lender prevents the lending rate from falling, and this effect does not approach to zero even with an increase in the number of lenders. Additionally, the low elasticity of the firm's demand function for debt also has the effect of increasing the maximum in the Bertrand model.

There are still several issues that remain to be addressed. One such issue is the heterogeneous lender differentiation, particularly by using different risk attitudes and incumbent loan portfolios. Additionally, further investigation into the impact of each parameter on the spread is essential.

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